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Narrow-Bandpass Waveguide Filters

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***Abstract*—A procedure is described whereby narrow-bandpass waveguide filters having ripple in both the passbands and stopbands can be synthesized in the form of coupled waveguide cavities. Orthogonal modes in square or circular waveguides are employed to enable negative coupling elements to be realized. As a consequence, very compact filters can be constructed. Experimental results on an 8-cavity orthogonal-mode narrow-bandpass filter are shown to agree well with theory.**

I. Introduction

IT HAS BEEN well known since the early work on filter synthesis by Darlington [1] and Cauer [2] that when frequency selectivity and bandpass loss are considered to be the important filtering properties, then the optimum filters are those exhibiting ripple in both passbands and stopbands.

However, the present design of narrow-bandpass waveguide cavity filters is largely based upon the work of Cohn [3], which realizes filters in the form of cascaded, synchronously tuned cavities exhibiting only monotonically increasing out-of-band attenuation. This restriction in filter design is principally due to the difficulty in transforming the optimum low-pass ladder networks to coupled-cavity structures.

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Kurzrok [4] describes how extra coupling applied between the first and last cavity of a direct-coupled 4-coaxial-cavity structure produces a zero of transmission in the stopband. Easter and Powell [5] describe similar filters in rectangular waveguide. Recently, Williams [6] has illustrated the realization of the fourth-order elliptic function in an orthogonal-mode circular-waveguide structure.

It is the purpose of this paper to extend this work by describing the waveguide synthesis of general filter functions having these optimum-amplitude filtering properties. Two structures which employ orthogonal-mode waveguide cavities are presented, and extensive use is made of the general coupling-cavity theory that is outlined by Atia and Williams [7]. An experimental 8-cavity circular-waveguide bandpass filter with eight poles and two zeros of transmission is shown to correlate well with theory.

II. Theory

An account of the equivalent circuit of generally coupled cavities was given by Reiter [8], who described how Maxwell's equations can be replaced by an equivalent infinite system of algebraic inhomogeneous equations. However, if the frequency band of interest is narrow, so that each cavity can be treated as a single resonant circuit [9] with multiple couplings to other

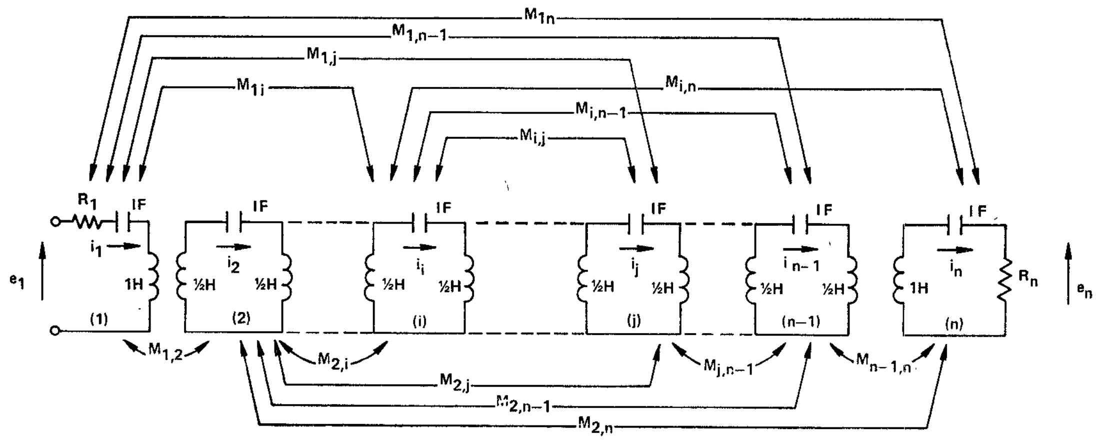


Fig. 1. Equivalent circuit of n coupled cavities.

cavities, then the equivalent circuit reduces to that shown in Fig. 1.

A general solution of such a narrow-band coupled-cavity structure has been presented in a paper by Atia and Williams [7], and, therefore, this section will only concentrate upon those aspects of the theory that are relevant to the synthesis of waveguide cavity filters. With reference to the equivalent circuit, the loop equations for narrow bandwidths can be written as:

$$\begin{bmatrix} e_1 \\ 0 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix} = \begin{bmatrix} S+R_1 & jM_{12} & jM_{13} & \cdot & \cdot & \cdot & \cdot & jM_{1n} \\ jM_{12} & S & jM_{23} & \cdot & \cdot & \cdot & \cdot & \cdot \\ jM_{12} & jM_{23} & S & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & S & jM_{n-1,n} \\ jM_{1n} & \cdot & \cdot & \cdot & \cdot & \cdot & jM_{n-1,n} & S+R_n \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ \cdot \\ \cdot \\ \cdot \\ i_{n-1} \\ i_n \end{bmatrix} \tag{1a}$$

or

$$E = Z \cdot \mathcal{I} \tag{1b}$$

where

$$S = j\left(\omega - \frac{1}{\omega}\right)$$

$$jM_{ij} \approx j\omega M_{ij} \approx j\omega_0 M_{ij}$$

$$\omega_0 = 1 \text{ rad/s}.$$

Further, the impedance matrix Z can be expressed in the form:

$$Z = (SI + M_R) \tag{2}$$

where

$$I = \text{the identity matrix}$$

$$M_R = R + jM$$

and the matrix R has all zero entries except for the (1, 1), (n, n) elements, which are R_1 and R_n, respectively. M is termed the coupling matrix and has general entries of M_{ij} for $i \neq j$, and 0 for $i = j$.

The voltage-transfer ratio of the coupled-cavity structure can be written in the form

$$\frac{i_n R_n}{e_1} = K[P(S)/Q(S)] \tag{3}$$

where K is a constant, $Q(S)$ is Hurwitz polynomial of degree n, and $P(S)$ an even polynomial in S whose degree is $\leq (n-2)$.

The general coupling theory [7] shows how the matrix M_R can be evaluated in terms of a given low-pass transfer function, $t(s)$[1] ($s = j\omega$), by first extracting the resistance terminations from the equation,

$$R_1 + R_n = \text{coefficient of } s^{n-1} \text{ term of the normalized denominator polynomial of } t(s). \tag{4}$$

Then, by following Darlington's procedure [1], the short-circuit input and transfer admittances can be obtained. The poles of the short-circuit admittances are the eigenvalues of the coupling matrix M. The first and last rows of a similarity transformation which gives the matrix M from its eigenvalues can be obtained from the residues of the short-circuit admittances. Since $P(S)$ is an even polynomial in S, it follows that the general short-circuit coupling matrix can be placed in the form:

[1] The low-pass to bandpass transformation is given by $s = j(\omega_0/\Delta\omega)(\omega/\omega_0 - \omega_0/\omega) = j(\lambda/\Delta\omega)$, where $\lambda = (\omega - 1/\omega)$.

$$M = \begin{bmatrix} 0 & M_{12} & 0 & M_{14} & 0 & M_{16} & \cdot & \cdot & \cdot & M_{1,n} \\ M_{12} & 0 & M_{23} & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & M_{23} & 0 & M_{34} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ M_{14} & 0 & M_{34} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ M_{16} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & M_{n-1,n} \\ M_{1,n} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & M_{n-1,n} & 0 \end{bmatrix} \tag{5}$$

The synthesis of bandpass filters in waveguide cavities is most conveniently accomplished by assuming a symmetrical network, i.e., $R_1 = R_n = R$, and that M is symmetrical about the antidiagonal (as well as about the main diagonal). Then it is advantageous to use the even-mode coupling matrix M_e of the network. (This matrix corresponds to the excitation of the unterminated network by two identical zero impedance voltage sources applied at both ends.) It can be readily shown that M_e represents an $n/2$ by $n/2$ matrix, whose elements are obtained by folding along the center line of the rows and columns of M.[2] M_e has the form:

i.e., first, an orthogonal T_e matrix is constructed by using the Gram–Schmidt orthonormalization process, and then a general even-mode coupling matrix M_e is evaluated, using the relation:

$$-M_e = T_e \Lambda T_e^t \tag{7}$$

where Λ is a diagonal matrix whose elements are equal to the even-mode eigenvalues.

Lastly, by applying a modification of Jacobi's diagonalization process, elements of the M_e matrix can be annihilated and the required cavity-coupling matrix can be formed.

$$M_e = \begin{bmatrix} M_{1,n} & M_{1,2} & M_{1,n-2} & M_{1,4} & M_{1,n-4} & \cdots & M_{1,n/2} \\ M_{1,2} & M_{2,n-1} & M_{2,3} & \cdot & \cdot & \cdots & \cdot \\ M_{1,n-2} & M_{2,3} & M_{3,n-2} & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdots & \cdot \\ M_{1,n/2} & \cdot & \cdot & \cdot & \cdot & \cdots & M_{n/2,(n/2)+1} \end{bmatrix} \tag{6}$$

The eigenvalues (λ_k) of the even-mode network are identical in magnitude to the $n/2$ distinct eigenvalues of the original network. Furthermore, a new orthogonal-transformation matrix T_e will exist in which only the first-row elements are known. These elements are equal to $\sqrt{2}$ times the $n/2$ distinct elements of the first and last rows of the full orthogonal-transformation matrix, and are obtained from the residues of the short-circuit input and transfer admittances.

Quite obviously, by working with the reduced matrices T_e and M_e, computation accuracy is greatly improved and computation time is significantly reduced. The procedure follows directly that described in [7];

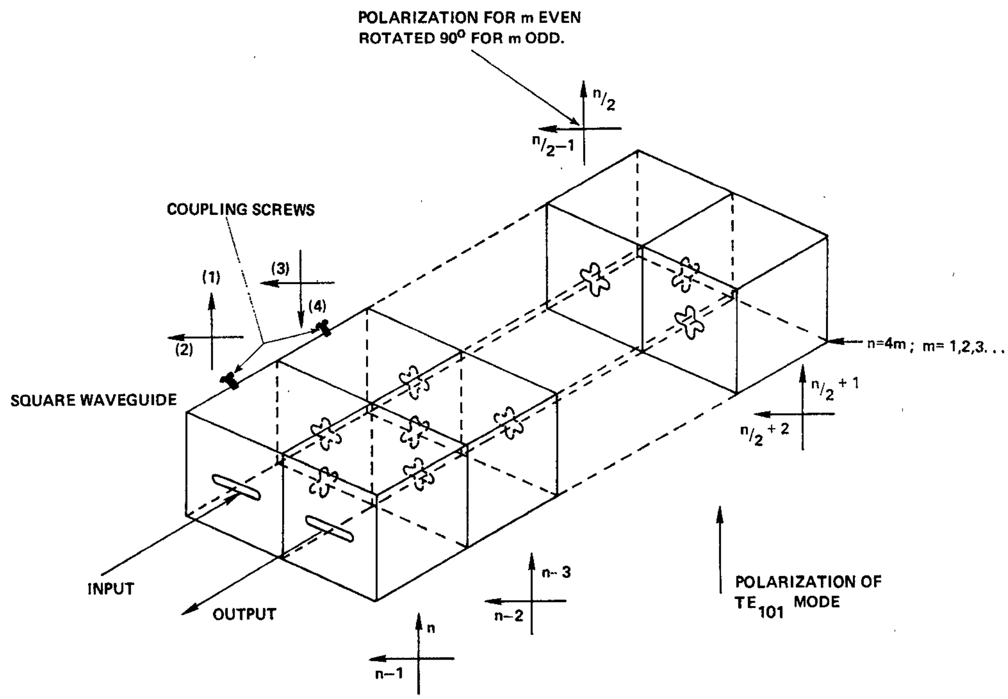
[2] It would be equally valid to use the odd mode. The only difference would be a sign change of the folded elements. The procedure to be described is illustrated for n even. However, the method follows in a nearly identical manner for n odd.

For narrow-bandpass waveguide filters having ripple in both passbands and stopbands the general low-pass power transfer function that can be synthesized in coupled cavities is:

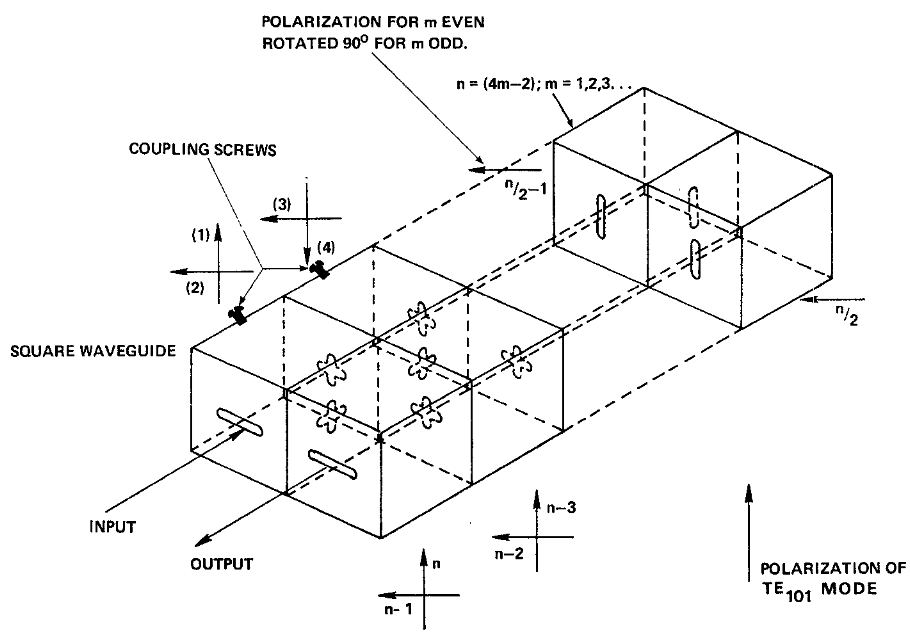
$$|t(s)|^2 = \frac{1}{1 + \epsilon^2 (-1)^r s^{2r} \dfrac{\prod_{k=1}^{m} (s^2 + z_k^2)^2}{\prod_{k=1}^{l} (s^2 + p_k^2)^2}} \tag{8}$$

where $r + 2m + 1 \geq 2l$.

The bandpass form of this function can be realized by the orthogonal-mode square-cavity structure shown in Fig. 2. For the particular case where $r = 2$ and $l \leq$ Int $[(2m+r)/4]$, $t(s)$ can be synthesized in a direct-coupled



(a)



(b)

Fig. 2. (a) A general coupled-cavity filter, $n=4m$, $m=1, 2, 3$. (b) A general coupled-cavity filter, $n=(4m-2)$, $m=1, 2, 3$.

square- or circular-orthogonal-cavity structure illustrated in Fig. 3. An extra cavity at the end of each of these structures enables odd-order transfer functions to be generated.[3]

This paper describes the synthesis procedure of an eighth-order nonequiripple function ($r=2$, $m=3$, and $l=2$) that can be realized in the structure shown in Fig. 3.

[3] The general odd-order equiripple filter functions for which $r+2m=2l$ cannot be synthesized in a simple coupled-cavity structure.

III. Eighth-Order Nonequiripple Filter Function

With reference to (8), the eighth-order function is:

$$|t(s)|^2 = \frac{1}{1+\epsilon^2 s^4 \dfrac{(s^2+Z_1^2)^2(s^2+Z_2^2)^2(s^2+Z_3^2)^2}{(s^2+P_1^2)^2(s^2+P_2^2)^2}}. \quad (9)$$

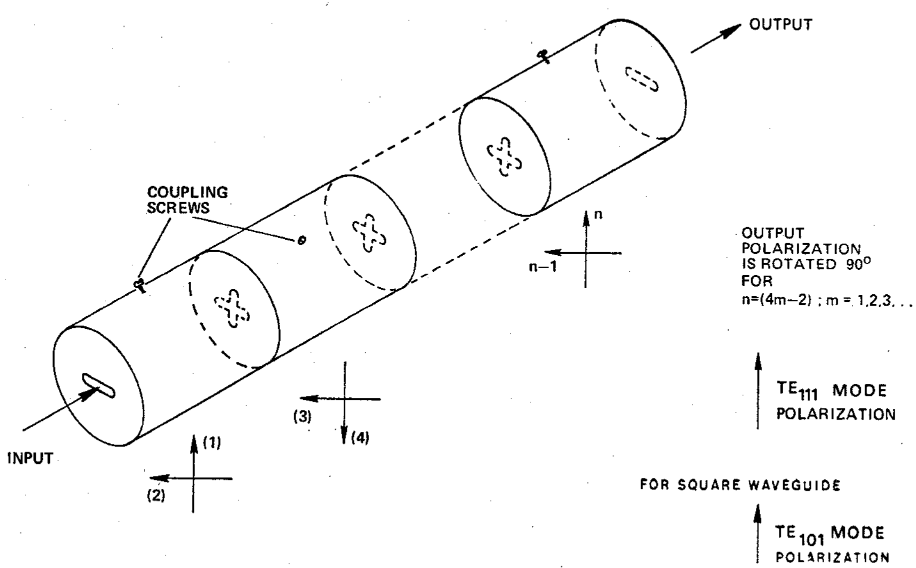


Fig. 3. Orthogonal circular-waveguide filter.

For a transmission loss of 0.05 dB at the band edge, characteristic function zeros given by

$$Z_1 = 0.00000 \qquad Z_2 = 0.737347 \qquad Z_3 = 0.973437$$

and poles given by

$$P_1 = 1.454154 \qquad P_2 = 1.919754$$

the ripple constant is $\epsilon = 13.459281$.

Equation (8) can be placed in the form $|t(s)|^2 = t(s) \cdot t(-s)$, and the low-pass voltage transfer function $t(s)$ can be extracted as:

$$t(s) = P(s)/[\epsilon \cdot Q(s)] \tag{10}$$

where

$$P(s) = s^4 + 5.800019s^2 + 7.793130$$

and

$$Q(s) = s^8 + 3.443494s^7 + 7.420055s^6 + 11.116641s^5 + 12.378520s^4 + 10.341950s^3 + 6.321218s^2 + 2.611730s + 0.579015.$$

With reference to (4), the normalized termination can be immediately evaluated as:

$$R_1 + R_6 = 3.443494 \tag{11}$$

i.e.,

$$R = 1.721747.$$

With R known it is now possible to evaluate the short-circuit input and transfer admittances of the network and the corresponding even-mode admittance. Expressed in terms of the bandpass variable λ, where $\Delta\omega = 1$,[4] the even-mode admittance is given by:

[4] It is most convenient to compute R and the coupling matrix M by setting $\Delta\omega = 1$, since $\Delta\omega$ acts as a scaling factor on these network parameters.

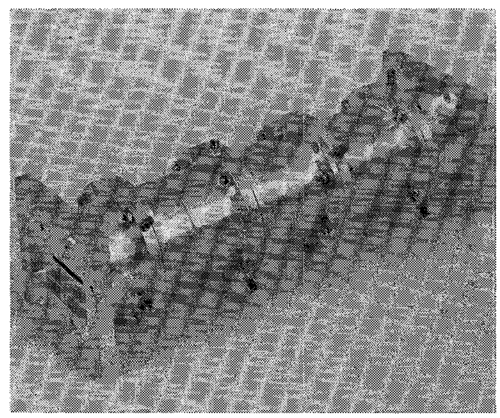


Fig. 4. Experimental 8-cavity circular-waveguide filter.

$$Y_e(\lambda) = \frac{C_{e11}}{\lambda - \lambda_{1e}} + \frac{C_{e12}}{\lambda - \lambda_{2e}} + \frac{C_{e13}}{\lambda - \lambda_{3e}} + \frac{C_{e14}}{\lambda - \lambda_{4e}} \tag{12}$$

where

$$\lambda_{1e} = -0.342867$$
$$\lambda_{2e} = 0.881291$$
$$\lambda_{3e} = -1.330643$$
$$\lambda_{4e} = 1.338208$$

and

$$C_{e11} = (0.49593)^2$$
$$C_{e12} = (0.37869)^2$$
$$C_{e13} = (0.56740)^2$$
$$C_{e14} = (0.53731)^2.$$

By recognizing that the residues (C's) are the squares of the first row of the orthogonal-transformation matrix T_e, and by using the Gram–Schmidt orthonormalization process with the vectors (C_{e11}, C_{e12}, C_{e13}, C_{e14}), (0, 1, 0, 0), (0, 0, 1, 0), and (0, 0, 0, 1) as a basis, the general transformation matrix T_e is obtained as:

$$T_e = \begin{bmatrix} 0.49593 & 0.37869 & 0.56740 & 0.53731 \\ -0.20292 & 0.92552 & -0.23216 & -0.21985 \\ -0.41581 & 0.00000 & 0.79003 & -0.45050 \\ -0.73484 & 0.00000 & 0.00000 & 0.67825 \end{bmatrix} \tag{13}$$

and the even-mode coupling matrix M_e becomes:

$$M_e = \begin{bmatrix} 0.00000 & 0.36060 & -0.84970 & 0.61263 \\ 0.36060 & 0.73375 & 0.34767 & -0.25067 \\ 0.84970 & 0.34767 & -0.61821 & -0.51365 \\ 0.61263 & -0.25067 & -0.51365 & 0.43046 \end{bmatrix}. \tag{14}$$

If all possible couplings could be realized, then M_e and R would represent a general solution of the eighth-order nonequiripple filter function. However, for realization of the transfer function in the waveguide structure shown in Fig. 3, it is necessary to reduce the couplings M_{16}, M_{25}, and M_{27} to zero. This is achieved by applying successively orthogonal similarity transformations, which annihilate M_{e13} $(=M_{e31})$, M_{e22}, and M_{e24} $(=M_{e42})$ to M_e.

Two solutions can be derived:

$$-M_e = \begin{bmatrix} 0.00000 & -1.09022 & 0.00000 & 0.19685 \\ -1.09022 & 0.00000 & -0.74917 & 0.00000 \\ 0.00000 & -0.74917 & -0.01081 & -0.53306 \\ 0.19685 & 0.00000 & -0.53306 & 0.55680 \end{bmatrix} \tag{15}$$

$$-M_e = \begin{bmatrix} 0.00000 & 1.09713 & 0.00000 & 0.15375 \\ 1.09713 & 0.00000 & 0.58529 & 0.00000 \\ 0.00000 & 0.58529 & -0.36627 & 0.41800 \\ 0.15375 & 0.00000 & 0.41800 & 0.91226 \end{bmatrix} \tag{16}$$

IV. Experimental Results

In the previous section, the coupling matrix M and the resistance termination R were evaluated for the eighth-order function given by (9). The circular-waveguide structure illustrated in Fig. 3 can be constructed to realize these parameters and, hence, the required transfer function. This structure employs orthogonal TE_{111} circular-cavity modes in four physical cavities, resulting in eight electrical cavities coupled in cascade. Further, additional coupling is provided between cavities one and four, three and six, and five and eight. From a physical point of view, it is these couplings which provide the zeros of transmission. Couplings M_{12} $(=M_{78})$ and M_{34} $(=M_{56})$ are provided by screws whose spacial orientation is arranged to provide M_{14} $(=M_{58})$ and M_{36} with the required signs. Couplings M_{23} $(=M_{67})$, M_{45}, M_{36}, M_{14} $(=M_{58})$, and the parameter R (which is the loaded Q of the input and output cavities) are realized by long thin coupling slots.

The design of these slots together with the cavity dimensions are described in [6] and [10]. This procedure was employed to design an 8-cavity filter with the coupling values of M_e given in (15) to operate with a bandwidth of 37 MHz at a center frequency of 3973 MHz. A photograph of the experimental filter is shown in Fig. 4, the return loss and amplitude response in Fig. 5, and the time delay in Fig. 6. It is important to note that the experimental results show excellent agreement with theory, and in particular the four zeros of transmission are present at their correct frequency positions in the stopband.

A swept frequency response of this filter between 3.7 GHz and 6.0 GHz has been made, and the transmission curve is shown in Fig. 7. Good correlation with the spurious higher order cavity modes, TM_{011} at 4.86 GHz, TE_{112} at 5.66 GHz, and TE_{211} at 5.80 GHz, is evident. It is important to note that because of the filter's geometrical symmetry, no circular TM_{010} mode is propagated (at least not above 70 dB) at 4.2 GHz through the filter. This type of spurious transmission response for the orthogonal-mode filter is similar to that shown by conventional waveguide designs.

The measured expanded in-band loss of the filter in Fig. 6 shows a center-frequency loss of 0.4 dB which corresponds to an average Q of 10 000. This loss is significantly less than that which could be achieved by a comparable Chebyshev or Butterworth waveguide filter (approximately 1.5 dB), and clearly demonstrates the low-loss properties of the near-optimum transfer function.

V. Conclusion

A method of synthesizing filter transfer functions having ripple in both their passbands and stopbands in

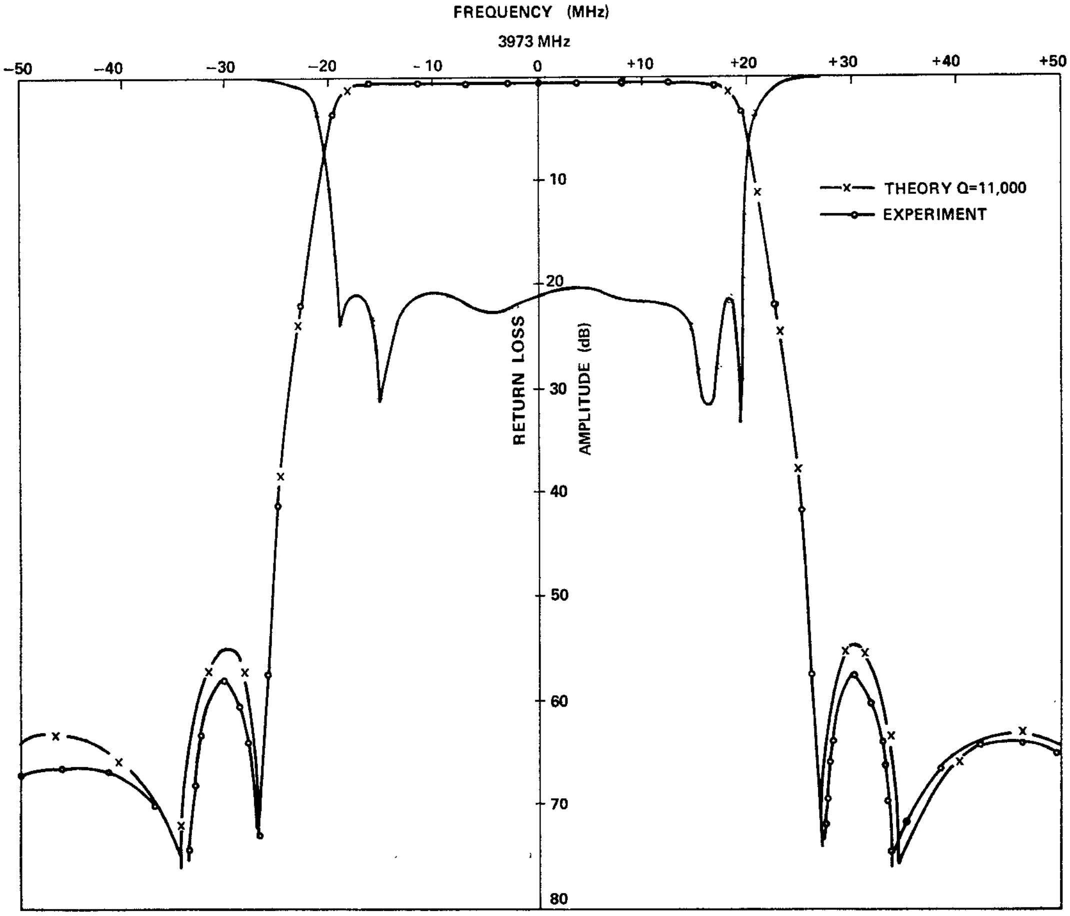


Fig. 5. Transmission response and return loss.

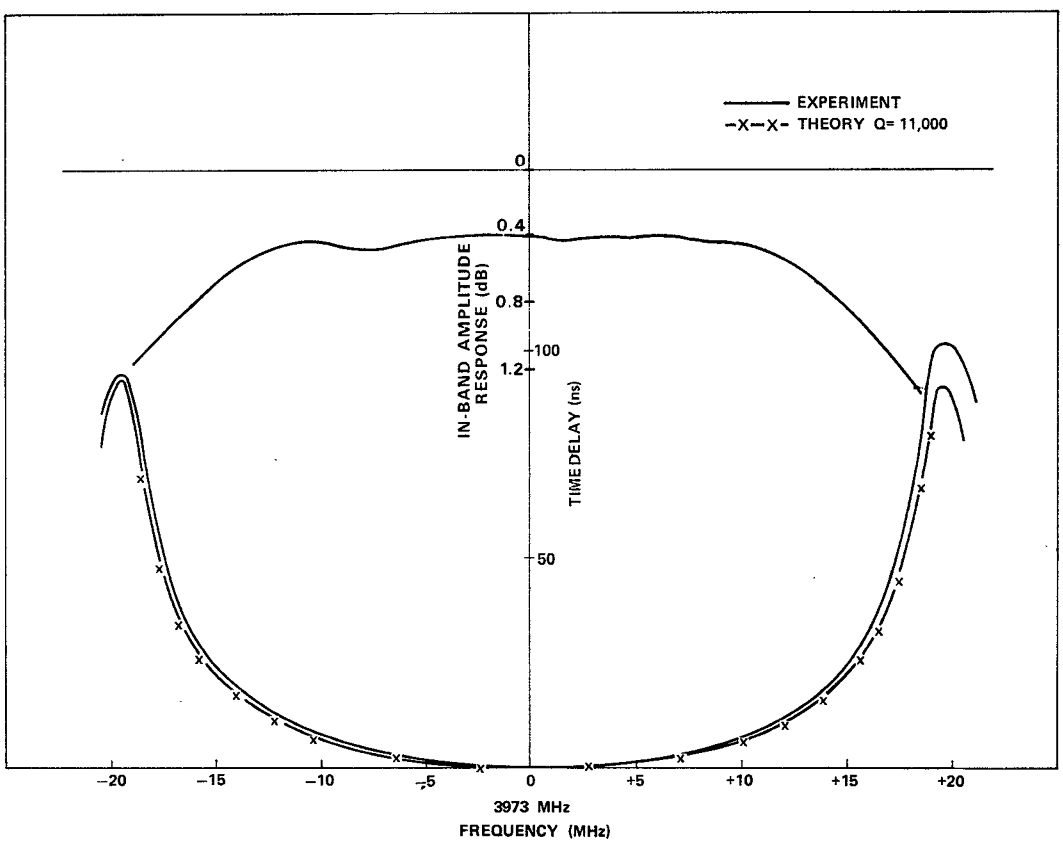


Fig. 6. Time delay and inband amplitude response.

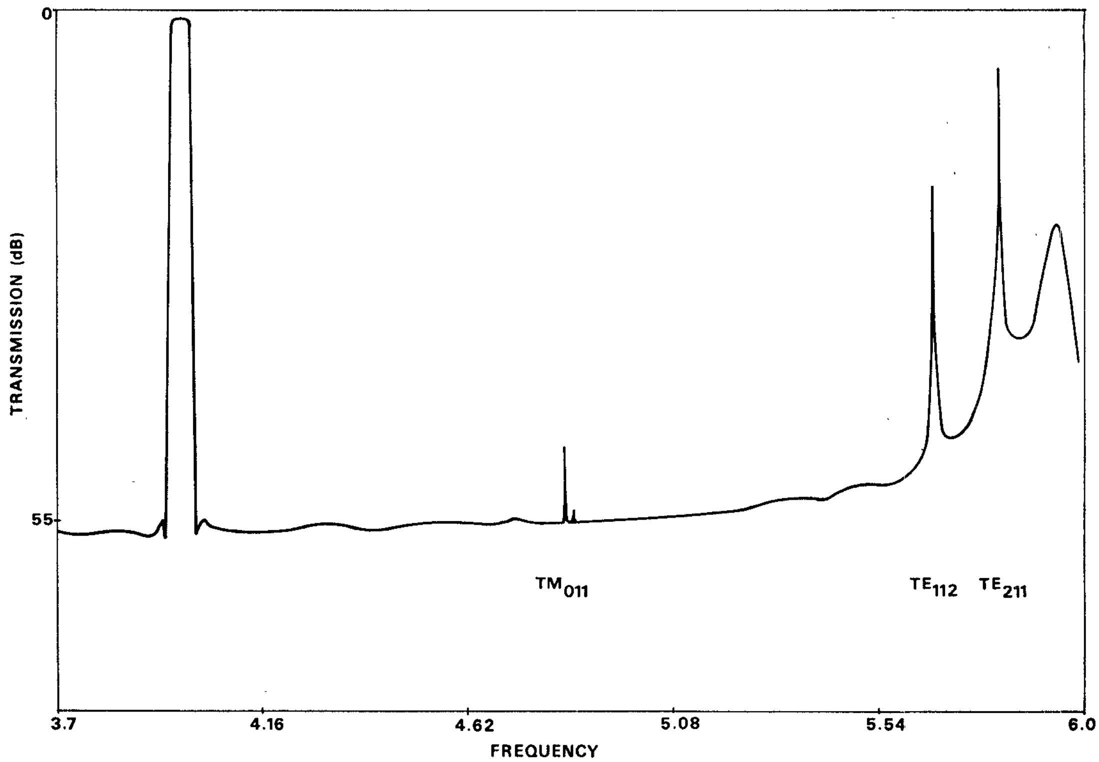


Fig. 7. Out-of-band transmission response.

the form of coupled waveguide cavities has been presented. By using orthogonal modes in circular- or square-waveguide cavities, both positive and negative coupling values can be realized. An 8-cavity filter was constructed and the experimental results obtained from it, including the spurious responses, agree well with theory.

The type of filter described has the advantage of having lower loss than a similar Chebyshev or Butterworth design, besides being approximately half the size and weight. It is interesting to note that Chebyshev and Butterworth filters can also be constructed by using the orthogonal-mode-cavity structure.

Acknowledgment

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